GEO 5500 Numerical Methods in the Geosciences
Computer Assignment #9
Boundary Value Problems:
Parabolic Equations

Assigned: March 24, 2005
Due: April [redacted], 2005
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Recommended reading: Lindfield and Penny, Chapter 6

This assignment will count as two regular computer assignments.

Consider the 1-dimensional advection-diffusion equation for a chemical constituent, C, with a constant concentration (which can represent contamination) of 100 at x = 0 m and concentration of 0 at x = 100.

$$\frac{\partial C}{\partial t} = -V\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2}$$

Using finite difference methods, this equation can be applied to a variety of environmental problems. You should begin this assignment by writing out the governing equation, the finite difference formulation, and the appropriate boundary and initial conditions on paper. This will guide you as you write your computer code. Assume a diffusion constant, D, of 10^{-6} m^2/s and velocity, V, of 10^{-7} m/s.

(a). Write a computer code of your finite difference formulation using time steps and grid spacings that are appropriate for the problem.

(b). Evaluate the finite difference stability criteria. Run the program using $(D\Delta t)/(\Delta x)^2$ < (less than) 1/2. What happens and why?

(c). Plot the concentration profiles at time intervals that allow you to see evolution of the concentration profile. (You can do this either with a 2-dimensional plot with various lines or as a 3-dimensional plot i.e., time-space-concentration).

(d). If you assume that the contamination started in 1970, use your code to calculate concentration profiles using a variety of values for the diffusion constant (actually hydrodynamic dispersion).

(e). If the contamination source was cleaned up in 1990 calculate what the concentration profile would look like today.

(f). Modify your 1-dimensional finite difference code so that you can include flux (Type II) boundary conditions at x=100 boundary.

(g). Write a separate program to solve the problem using an implicit finite difference scheme.