

Some Class Notes for Geothermal Systems for Geoscientists GEO 5920/6920/692 sec 03

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1 Basic Concepts & Equations

To start, let us define some simple quantities and nomenclature. There is an effort to have consistent variables throughout this document (x is always a spatial dimension, T for temperature, etc.), but that is not totally guaranteed - search back from an equation to find undefined terms!

1.1 Heat

Recall *heat* is a type of internal energy (average particle velocity for a gas, vibrational energy for a solid, etc.). We can compute the amount of heat required to change the temperature of a system from:

$$Q = mc\Delta T \tag{1}$$

where Q is the heat (in joules, J), m is the mass of the system (in kg), c is the specific heat (an empirical material property, in J/(kg K)), and ΔT is the temperature change (in kelvin, K or °C). In many cases, we have a known volume of material at some density, in which case we can replace the mass by the density, ρ , times volume, V :

$$Q = V\rho c\Delta T$$

1.2 Power

We are often concerned with transient (time-varying) systems. In particular, we will often be concerned with *power* which is energy/time. We can make our heat content equation transient by taking the time derivative (expressed by a dot):

$$P = \dot{Q} = \frac{dQ}{dt} = \frac{d}{dt}(mc\Delta T) \quad (2)$$

where P is power (J/s or watts, W).

Now, when we apply the time derivative to both sides, we get to choose what we hold constant and what varies on the right. So, we can say that temperature varies over time to get:

$$P = mc\dot{T}$$

or, we can choose to let the mass vary over time for a fixed temperature drop (think of steam flowing out from a pipe):

$$P = \dot{m}c\Delta T$$

From this latter form, we can recast to a one suitable for use with a known area and velocity of flow:

$$P = Av\rho c\Delta T$$

where A is the area of the flowing material (area of pipe, channel, etc.), v is the velocity of flow, and ρ is the density of the fluid.

1.3 Latent Heat & Phase Transitions

Heat is also absorbed or released by phase transitions (solid to liquid, liquid to gas, etc.) without changing temperature. The heat involved in a phase transition can be computed from:

$$Q = \Delta mL \quad (3)$$

where Q is the heat (J), Δm is the mass **undergoing transition** (kg), and L is the latent heat for the transition (fusion for solid-to-liquid, vaporization for liquid-to-gas) in J/kg. Latent heat is a material-specific property which can be found in thermophysical property references. Also recall that for

multi-component systems, we just superimpose each component to get the total heat change:

$$Q = \sum_{i=1}^N \Delta m_i L_i \quad (4)$$

This is an example of one of the most important concepts in physics: superposition - the case where a total system behaves as the sum of its parts. This is one of the defining characteristics of a linear system, and a major reason we will always try to use linear systems in geothermics.

1.4 Pressure

Pressure is force over area, and is computed like gravitational potential energy:

$$P = \rho gh \quad (5)$$

where P is pressure (Pa), ρ is density (kg/m^3), g is the acceleration due to gravity (typically $9.8 \text{ m}/\text{s}^2$), and h is the height of the column (m). We are computing pressure due to a column of mass, which is typically water.

So, we typically compute *hydrostatic* pressure (pressure due to a column of water) or *lithostatic* pressure (pressure due to a column of rock). Both pressures are computed using the same equation, but the density (ρ) changes from ~ 1000 for water to (typically) 2500 to 2800 for rock.

Note that when computing the absolute pressure at some depth, we will typically need to add the pressure of the atmosphere ($\sim 1 \text{ bar}$), unless the system is somehow completely independent of the atmosphere.

1.4.1 Absolute vs. Gauge Pressure

Absolute pressure is the actual pressure, due to all factors. Gauge pressure is the pressure above or below atmosphere. Hence, gauge pressure can be negative, but absolute pressure really won't. For most calculations, absolute pressure is needed, but most measurements are in gauge pressure; to convert, add the atmospheric pressure to gauge pressure to get absolute. Atmospheric pressure is typically $\sim 1 \text{ bar}$.

1.5 Enthalpy

Enthalpy is a measure of the usable energy of a fluid - how much work a mass of fluid can perform. Enthalpy includes energy from temperature, pressure, and phase changes. Hence, the enthalpy of steam at a given temperature is quite a bit more than the enthalpy of liquid water at the same temperature. Enthalpy is defined as:

$$h = U + PV \quad (6)$$

where U is the internal energy (J/kg) due to phase changes, temperature, etc. P is the pressure and V is the volume; the product is the energy required to make space for the fluid in ambient conditions. Thus, the enthalpy of a fluid can be increased by increasing ambient pressure without changing temperature or state.

To compute the internal energy, we typically can use the heat content and latent heats:

$$U = mc\Delta T + \Delta mL_v$$

where ΔT is a temperature change; from 0 K if computing absolute enthalpy, or from the initial temperature if computing enthalpy change. Δm is the mass that has vaporized and L_v is the heat of vaporization, as most enthalpy calculations are for steam & water. For supercritical steam, the internal energy is more complex. In practice, enthalpy is empirically measured at various pressures, temperatures, and states.

When computing available energy, it is easiest to compare the enthalpies of the initial and final states, including appropriate phases (conditions). The difference in enthalpy gives the usable energy for work; thus commercial power plants condense steam, which releases far more energy than liquid cooling.

1.6 Supercritical Conditions

Water taken above 220.64 bars (22.064 MPa) and 373.95 °C becomes supercritical steam, which acts a little like steam and a little like liquid water. Thus, the boiling point vs. depth curve for water (saturated steam table) ends at 373.95 °C and 220.64 bars. Higher temperatures and pressures requires the use of supercritical steam values, which have been measured. See the NIST water properties tables on the WWW.

2 Equations from Thermal Physics

2.1 Advection-Diffusion Equation

We start by considering a test volume (δV) and imposing conservation of energy, which allows us to write the basic form of an advection-diffusion equation:

$$-\vec{\nabla} \cdot \vec{q} + A - \phi \rho_w c_w \vec{v} \cdot \vec{\nabla} T = \rho c \frac{\partial T}{\partial t} = \rho c \dot{T} \quad (7)$$

where \vec{q} is the heat flux (flow), A is the (constant) heat production, ϕ is the porosity, ρ_w is the density of water, c_w is the specific heat of water by mass, \vec{v} is the advection velocity, T is temperature, ρ is the density of the rock, and c is the specific heat of rock by mass.

In conceptual terms, our advection-diffusion equation states that the divergence of the heat flow (point sources) plus heat production minus the heat transported by moving material equals the temperature change with time. The material constants (ρ , c , ϕ , etc.) are present to balance units.

2.2 Fourier's Law & Heat Equation of Geothermics

We also recall Fourier's Law, which relates flux to gradients:

$$\vec{q} = -k \vec{\nabla} T \quad (8)$$

where \vec{q} is the heat flow, k is the (constant) thermal conductivity, and T is temperature.

Substituting Fourier's Law into the advection-diffusion equation, we can get the basic heat equation for geothermics:

$$\vec{\nabla}^2 T + \frac{A}{k} - \frac{\phi \rho_w c_w}{k} \vec{v} \cdot \vec{\nabla} T = \frac{\rho c}{k} \dot{T} = \frac{1}{\alpha} \dot{T} \quad (9)$$

where α is the (thermal) diffusivity: $\alpha = k/\rho c$.

2.3 1-D Geothermics

In one dimension, our geothermics equation reduces to:

$$\frac{\partial^2 T}{\partial z^2} + \frac{A}{k} - \frac{\phi \rho_w c_w}{k} v_z \frac{\partial T}{\partial z} = \frac{\rho c}{k} \frac{\partial T}{\partial t} = \frac{1}{\alpha} \dot{T} \quad (10)$$

This equation can be greatly simplified depending on the type of system under study. For steady-state systems, $\dot{T} = 0$; for systems with no heat production, $A = 0$; for conductive systems, $\vec{v} = 0$. Thus, a 1-D conductive, steady-state system without heat production reduces to:

$$\frac{d^2T}{dz^2} = 0 \quad (11)$$

which is the classic Poisson's Equation (in 1-D).

Adding heat production gives a classic conductive geotherm equation:

$$\frac{d^2T}{dz^2} + \frac{A}{k} = 0 \quad (12)$$

which has a solution (by integration) of:

$$T(z) = T_0 + \frac{q_0}{k}z - \frac{Az^2}{2k} \quad (13)$$

Transient systems, without heat production:

$$\frac{\partial^2T}{\partial z^2} = \frac{\rho c}{k} \frac{\partial T}{\partial t} = \frac{1}{\alpha} \dot{T} \quad (14)$$

which is the classic diffusion equation without sources.

2.4 Advection

Steady-state systems without heat production, with advection (water flow):

$$\frac{\partial^2T}{\partial z^2} + \frac{\phi\rho_w c_w}{k} v_z \frac{\partial T}{\partial z} = 0 \quad (15)$$

Assume a system with vertical flow in the range $z = 0$ to $z = L$. For combined boundary conditions (constant temperature, T_0 , at top; constant flux, q_L , at bottom), the system has a solution of:

$$T(z) = T_0 + \frac{\Gamma_L L}{\beta} \cdot \frac{\exp(\frac{\beta z}{L}) - 1}{\exp(\beta)} \quad (16)$$

where

$$\beta = \frac{\phi\rho_w c_w v_z L}{k_{eff}}$$

is the Peclet Number, and

$$\Gamma_L = \frac{q_l}{k_{eff}} = \left. \frac{\partial T}{\partial z} \right|_{z=L}$$

2.4.1 Thin Aquifer Flow

In geothermal systems, we often encounter horizontal flow in a thin aquifer. The temperature (and thus heat flow) fields have been addressed in a number of papers. Here, the results of Ziagos & Blackwell (1986) are reproduced. Assume a simple 2-D $((x, z), z \text{ positive down})$ system with a near-horizontal aquifer with flow from $x = 0$ to $x = \infty$; the aquifer is fed from a vertical source at $x = 0$ with constant temperature T_a . The aquifer is capped by an impermeable layer of thickness l with a free surface at $z = 0$ at fixed temperature $T_s = 0$. Below the aquifer is an impermeable half-space. Rock and fluid thermophysical properties are constant. We assume a “thin” aquifer, which comes down to allowing an assumption that the aquifer is isothermal across its thickness. In the paper, the aquifer is assumed to be zero thickness to simplify the math, but the results hold if the aquifer is of finite thickness and isothermal vertically. So, we assume a finite aquifer thickness, h , and vertically isothermal aquifer.

For a steady-state solution, it can be shown that the half-space below the aquifer ($z > l + h$) is a conductive geotherm with boundary temperature at $z = l$ depending on x :

$$T(x) = T_a \exp\left(-\frac{\gamma x}{l}\right) \quad (17)$$

where T_a is the temperature of the fluid at input ($x = 0$) and

$$\gamma = \frac{k}{\phi \rho_w h c_w v}$$

for the thermal conductivity of the rock (k), heat capacity of the fluid (c_w), velocity (v), porosity (ϕ), fluid density (ρ_w), and aquifer thickness (h).

Similarly, it can be shown that the steady-state surface heat flux due to aquifer flow is:

$$q(x)|_{z=0} = q_0 \exp(-\gamma x/l) \quad (18)$$

where q_0 is set by the initial aquifer temperature:

$$q_0 = \frac{kT_a}{l}$$

In many cases, a transient situation is encountered, as the steady-state solution is only valid on extremely long time-scales (e.g. multiple millions of years). For geothermal systems, the lifetime of the system is typically

not long enough to reach steady-state in this model system. Approximate solutions for “short-time” and “long-time” conditions can be computed using Laplace Transforms, as detailed in the paper.

Here, we use “short-time” to mean conditions when the initial flow front is traversing the system; for natural geothermal systems, we will almost never encounter these conditions. Natural systems are discovered while already flowing, which means we want the “long-time” conditions - flow has progressed for some time, but not long enough for steady-state to be a good approximation. “Short-time” solutions are given in the paper, but are not reproduced here for space. The “long-time” solutions (see Ziagos & Blackwell, 1986, appendix B for derivations) are:

$$T(x, z, t) = T_a \exp(-\gamma x/l) \sum_{n=0}^{\infty} \left[\operatorname{erfc} \left(\frac{(2n+1)l - z + \gamma x}{(4\alpha t'')^{1/2}} \right) - \operatorname{erfc} \left(\frac{(2n+1)l + z + \gamma x}{(4\alpha t'')^{1/2}} \right) \right] \quad (19)$$

for the temperature in the upper layer. In the aquifer, we have:

$$T(x, z, t) = T_a \exp(-\gamma x/l) \operatorname{erfc} \left(\frac{\gamma x + z - l}{(4\alpha t'')^{1/2}} \right) \quad (20)$$

And in the half-space below the aquifer, temperatures are:

$$T(x, z, t) = T_a \exp(-\gamma x/l) \operatorname{erfc} \left(\frac{\gamma x}{(4\alpha t'')^{1/2}} \right) \quad (21)$$

In all three equations, t'' is computed from:

$$t'' = t' - \frac{\gamma l x}{3\alpha} = t - \frac{x}{v} - \frac{\gamma l x}{3\alpha}$$

3 Geophysics for Geothermal Systems

Here we will look at some examples of using geophysics to find and monitor geothermal systems. We start with the simplest, gravity.

3.1 Gravity

In the context of geophysics, we use the term "gravity" to refer to the acceleration due to gravity on the surface of the Earth.

The force of gravity can be calculated from Newton's law for gravitation:

$$F_{grav} = G \frac{Mm}{r^2}$$

for F_{grav} the force due to gravity acting on both masses, G the universal gravitation constant ($6.673 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$), M and m the two masses under consideration, and r the distance between the masses. For geophysics, we are concerned with the force of gravity on the surface of the Earth, so M is the Earth's mass, and we want the acceleration due to gravity (or the force on a unit test mass), so we get the basic gravity equation:

$$g = \frac{F_{grav}}{m} = \frac{GM}{r^2}$$

For the Earth mass, radius, and gravitation constant, the average acceleration is $\sim 9.8 \text{ m/s}^2$.

The Earth is not a sphere, and it spins, so g is not constant across the surface. In fact, the variation is up to about 5%.

The SI unit for gravity acceleration is m/s^2 , but the typical changes are very small, making m/s^2 impractical. Instead, we use the older cgs unit of the gal, named in honor of Galileo. A gal is defined as 1 cm/s^2 , so the average gravity is $\sim 980 \text{ Gals}$. In practice, most gravity changes and signals are measured in milligals (mGal) or microgals (μGal). Thus, the average gravity is $\sim 980\,000 \text{ mGal}$. Typical geophysical signals are 0.01 to 1000 mGal.

Gravity can be used to infer 2 different pieces of information of interest to geothermal systems: (1) geologic structure, as inferred from lateral density contrasts, or (2) reservoir changes, as inferred from temporal changes in gravity. Thus, gravity is normally interpreted as changes from some expected/model/baseline value in space and/or time.

Exploration gravity deals with changes in space of the absolute gravity values. The gravity field at a point is dependant on the distance to the Earth's center of mass and the density structure between the observation point and the center of mass. Thus, lateral changes in the gravity field reflect differences in elevation and density. If the elevation is known from other sources, the absolute gravity field can be predicted from theoretical formula.

The difference between observed and predicted gravity (the anomaly) is interpreted for density variations from the prediction model, which are interpreted as geologic structure. The most common anomaly used for exploration is the Complete Bouguer Anomaly, which accounts for the latitude of the observation point (Earth's rotation), ellipsoidal shape of the Earth, topography, elevation, and predicted average structure.

Gravity monitoring consists of repeated measurements over a fixed station network to look for gravity changes in time. Gravity changes are caused by changing topography, elevation, or density. Topography changes (e.g. construction) are better measured by other means, so are removed by modeling. Elevation and density changes are both of interest for geophysics; modern campaigns typically use high-precision GPS to monitor elevation changes and correct gravity changes. Density changes are interpreted as due to changes in reservoirs, such as steam, groundwater, or petroleum changes due to production, injection, natural recharge, etc. Note that changes are typically in the 10-1000 μGal range for monitoring, rather than mGal range for exploration.

Gravity is measured by various types of gravimeters. Most relative instruments (measure gravity increase or decrease) use a mass on a spring and measure the extension of the spring; longer extension corresponds to increased gravity. Absolute measurements used to be based on pendulum systems - the period of a pendulum is related to the value of gravity by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Modern absolute gravimeters measure the acceleration of a falling test mass in a vacuum using a laser interferometer. They are accurate to 1-5 μGal but require 1-24 hours of measurements.

A simple, but useful, interpretive equation is for an infinite horizontal slab, also known as the Bouguer slab. Although calculated for an infinite slab, Bouguer slab calculations are very accurate for any case where the "slab" is more than 20x larger than the depth to the top of the slab. For a given height (h) and density (contrast) (ρ), the gravity effect of a Bouguer slab is:

$$\delta g = 2\pi G\rho h$$

Two useful rules of thumb: (1) 1 km rock @ 100 kg/m³ = 4 mGal and (2) 1 m water @ 1000 kg/m³ = 40 μGal = 0.040 mGal.

Another useful, relatively easy interpretative technique is to compute an effective excess mass due to gravity changes or anomalies. From Gauss' Theorem, the surface integral of an anomalous potential field can be used to compute the enclosed source; for gravity, the source is the excess mass (compared to a zero background field/anomaly).